A TRANS-TEMPORAL APPROACH TO THE LAWS OF PHYSICS

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ABSTRACT

We present a basic formulation of classical and quantum mechanics in which these two theories take on a similar form. In this formulation, classical mechanics becomes a non-deterministic theory, and quantum mechanics becomes capable of modeling the observable physical world as an objective reality. We use findings from the study of deterministic chaos to illustrate the extreme indeterminism that this formulation of classical mechanics entails for some systems, and we also show how the quantum theory displays corresponding indeterminism when applied to these systems.

Statistical laws are introduced in a uniform way into both the classical and the quantum mechanical models. We show that by making simple modifications of these laws, it is possible for both models to predict teleological phenomena of the kind observed in some parapsychological studies. We also show that both models entail non-local phenomena similar to the EPR effect, and we show that the models are consistent with the special theory of relativity.

1. INTRODUCTION

The rise of classical mechanics marked the culmination of a major shift in Western thinking from the Aristotelian conception of the world as an organism to the conception of the world as a clocklike mechanism operating according to mechanistic laws.[1] Classical mechanics is based on equations of motion which are deterministic; given the exact position and momentum coordinates of a classical system at time t_0, it is possible in principle to calculate the exact state of the system at any time in the future or the past. This property has many profound philosophical implications. For example, if nature is indeed strictly governed by such deterministic laws, then sentient beings must be pure machines, devoid of free will (or else "free will" must be defined as the deterministic interaction of particles in the brain).

With the later development of quantum mechanics, physics ceased to be strictly deterministic, and many physicists have tried to reintroduce the idea of conscious volition into our physical world view.[2-5] However, the advent of quantum mechanics did more than simply stress the role of the observer and add an element of indeterminism. In its standard formulation, quantum mechanics requires us to renounce the idea of forming a coherent theoretical picture of objective reality. Rather, it enjoins us to see physical theories merely as computational systems designed to predict patterns of statistical correlation in observed data.[6,7]

In this paper we present a reformulation of both quantum mechanics and classical mechanics which brings the two theories closer together, adding an element of indeterminism to classical mechanics, and also showing how quantum mechanics can be seen to provide an objective model of the world of our experience. This reformulation is based on the idea that the laws of physics should be seen as global statements about the history of events in space-time as a whole, rather than as relations of cause and effect governing the unfolding of events with the passage of time. Its starting point is Hamilton's principle in classical mechanics and the Feynman path integral in quantum mechanics, both of which present the laws of physics from a global viewpoint.

Using these starting points, we build up the idea of a process of trans-temporal selection which picks out a particular history of macroscopic events in space-time that conforms with the classical or quantum mechanical laws of nature. This process generates a specific space-time history which can be regarded as a representation of objective reality. However, although the process acts in accordance with the physical laws, it is not rigidly determined by them, and thus the laws do not precisely specify this history.

In Section 2 we review Hamilton's principle and introduce the process of trans-temporal selection for classical physics. In the next section we draw from the theory of deterministic chaos to illustrate the high degree of indeterminism allowed by our reformulation of classical mechanics. In Section 4 we show how statistical laws can be incorporated into this fomulation, and we discuss the definition of the arrow of time.

In Section 5 we discuss the various interpretations that have been given to the formalism of quantum mechanics. This is followed in the next two sections by the introduction of the quantum mechanical form of the process of trans-temporal selection. In Section 8 we argue that this process is compatible with special relativity, and we discuss the EPR effect and the idea of nonlocal interactions. In Section 9 we argue that the process of trans-temporal selection leads to certain predictions which seem to be confirmed by some of the data gathered in recent years in studies of psychokinesis. Finally, in Section 10 we make some concluding remarks and mention some avenues of future research.

2. CLASSICAL MECHANICS

We will begin our discussion of classical mechanics by defining Hamilton's principle for a simple classical system consisting of n particles of mass m with coordinates $\overline{Q}$ = $(Q^1,...,Q^{3n})$. Consider an arbitrary path Q(t) defining the history of the system from time $t=t_a$ to time $t=t_b$, and constrained by the requirement that $\overline{Q}(t_a)=\overline{Q}_a$ and $\overline{Q}(t_b)=\overline{Q}_b$. We can define a quantity called the action which is a global property of this entire history of events:

$$A(\bar{Q}) = \int_{t_a}^{t_b} L(\bar{Q}(t),\dot{\bar{Q}}(t),t)\ dt \qquad (1)$$

where the Lagrangian $L(\bar{Q},\dot{\bar{Q}},t)$ expresses the laws of physics for the system. The action A will generally vary as we vary the events in our arbitrary path. But if the action is invariant under all infinitesimal changes in the path between times t_a and t_b, then it turns out that this path satisfies the classical equations of motion.

We can define the sensitivity of the action to variations in the path as

$$S(\bar{Q}) = \int_{t_a}^{t_b} \left[\sum_{i=1}^{3n} |\delta A(\bar{Q})/\delta Q^i(t)|^2 \right]^{1/2} dt \qquad (2)$$

This quantity is simply a measure of how closely the path follows the classical equations of motion. Thus, for a system with $L=m\dot{\bar{Q}}\cdot\dot{\bar{Q}}/2-V(Q)$ the sensitivity is

$$S(\bar{Q}) = \int_{t_a}^{t_b} dt\,|m\ddot{\bar{Q}}(t)+\nabla V(\bar{Q}(t))| \qquad (3)$$

In the usual formulation of classical mechanics, only paths from $\bar{Q}_a$ to $\bar{Q}_b$ for which the sensitivity S=0 are regarded as having physical significance. We might ask, however, whether or not paths with S nearly equal to zero might also be significant. It turns out that for a broad set of classical systems there is a very large set of paths for which the sensitivity is very close to zero. Indeed, many of these paths differ greatly from the classical solution, but come so close to following the classical laws of motion that no experimental measurements occurring within the system could show any deviation from these laws.

This suggests a new way of looking at the classical laws of motion. We can regard them as defining a large class of paths from $\bar{Q}_a$ to $\bar{Q}_b$ which come so close to satisfying these laws that we could not hope to measure their deviation from them. We can call this the set of near-classical paths, and define it as

$$NC(\epsilon) = \{\text{Paths } \bar{Q} \text{ from } (t_a,\bar{Q}_a) \text{ to } (t_b,\bar{Q}_b) \mid S(\bar{Q})<\epsilon\} \quad (4)$$

where $\epsilon>0$ is very small. We postulate that the classical laws require the path followed in nature to lie in the set $NC(\epsilon)$, but that they do not specify which particular member of $NC(\epsilon)$ this path will be.

We can imagine that this path is selected in nature by an agency which conforms with the laws, but nonetheless is not rigidly constrained by them. We will refer to the action of this agency as the process of trans-temporal selection. We argue in later sections that various statistical regularities observed in nature can be accounted for by means of a probabilistic definition of this process. In this paper, however, we will not try to specify the exact nature of the agency that lies behind the selection process.

3. DETERMINISTIC CHAOS

In order to see the significance of this way of looking at the classical physical laws, it is necessary for us to get some idea of the nature of the near-classical paths. We can do this by examining some of the results found recently in the investigation of what is called deterministic chaos.

The term deterministic chaos refers to the observation that many classical systems respond with such sensitivity to small variations in position and momentum that their future behavior is chaotic and unpredictable in practice, even though it is exactly predictable in principle. Suppose $\bar{Q}$ is a path that exactly satisfies the equations of motion. Suppose that $\bar{Q}'$ is a path obtained by following $\bar{Q}$ up to time t, making very slight and gradual changes in some of the system's position and momentum variables over the interval from t to $t+\Delta t$, and letting the system evolve according to the equations of motion after $t+\Delta t$. Then, $\bar{Q}'$ nearly satisfies the laws of motion (so that it is a near-classical path), and it agrees with $\bar{Q}$ initially. But it may be that $\bar{Q}'$ greatly differs from $\bar{Q}$ shortly after $t+\Delta t$.

We can get a quantitative idea of the meaning of the terms "nearly" and "greatly" by considering some specific examples. We will begin by discussing a simple example of a chaotic system that was introduced and extensively studied by Henon.[8] Consider the two-dimensional potential

$$V(x,y) = (x^2+y^2)/2 + x^2y - y^3/3 \qquad (5)$$

Henon and his coworkers have shown that the classical motion of a particle in this potential can be very hard to predict when the energy of the particle is above a certain level. This is illustrated in Fig. 1 for a particle of mass m=1 gram and E=1/6 (using cgs units). The two-dimensional region in the figure represents a "surface of section" through the four-dimensional phase space of the system, and the many isolated points represent passages of a single particle trajectory through this surface.

As time passes, these points jump around on the surface of section in an apparently random fashion, and it appears that they will fill up a broad area in the limit as time goes to infinity. (In contrast, the points will remain confined to a single smooth curve in a "well behaved" classical system.)

This apparently random behavior of the particle's deterministic trajectory is evidently due to the fact that extremely small variations in the position or momentum of the particle will tend to be quickly amplified to a large degree. In fact, this amplification is so great that variations in position and momentum of the kind specified by the Heisenberg uncertainty principle can quickly give rise to macroscopic effects, even in the case of a particle of mass m=1 gram. To show this we have calculated a measure of amplification for a particular trajectory.

Let $\bar{x} = (p,q)$ represent a point in the 4-dimensional phase space of the system, and let $T_t(\bar{x})$ designate the phase space position at time t of a particle of mass m=1 that started at $\overline{x}$ at time t=0, and moved according to the Henon potential. If $\overline{\xi}$ is very small, then

$$T_t(\overline{x}+\overline{\xi}) - T_t(\overline{x}) \approx M(t,\overline{x})\overline{\xi} \qquad (6)$$

where $M(t,\overline{x})$ is the Jacobian matrix of the transformation T_t at $\overline{x}$. The magnitudes of the entries in $M(t,\overline{x})$ thus provide a measure of the deviation of $T_t(\overline{x}+\overline{\xi})$ from $T_t(\overline{x})$ caused by a small variation $\overline{\xi}$ in $\overline{x}$.

Fig. 2 depicts a closed orbit of period τ=34.86 sec for which $M(\tau,\overline{x}_0)$ can be readily calculated for a starting point $\overline{x}_0$. $M(\tau,\overline{x}_0)$ contains entries with magnitudes as large as 10^3, and its 4th and 8th powers contain entries with magnitudes of 4.2×10^{12} and 2.7×10^{26}. Suppose that the path $\overline{Q}$ is taken to be this orbit for the 1 gram particle, and that a variation in position or momentum of order 3.7×10^{-27} is made in the interval from t to t+Δt. If the variation is directed properly, the corresponding path $\overline{Q}'$ will deviate substantially from $\overline{Q}$ after a time of 8τ. For still later times the path $\overline{Q}'$ will not even resemble the closed orbit $\overline{Q}$.

According to the Heisenberg uncertainty principle, two conjugate position and momentum coordinates must have inherent uncertainties Δq and Δp with $\Delta q\Delta p \geq \hbar/2 \approx 5\times10^{-28}$. Here, however, we find that we need to know positions and momenta with greater accuracy than this to determine the classical trajectory over relatively short periods of time. Thus this system is so sensitive to small changes in initial conditions that to analyze it we must take quantum mechanical uncertainty into account.

In a system with extremely sensitive dependence on initial conditions, it is possible for the classical path followed by the system to take on a great variety of complex forms. To indicate how great this variety can be, let us consider a simple example based on an old fashioned pinball machine.

Consider an infinite array of fixed disks of radius r, centered on the lattice points (x,y) of the plane. Assume that another disk of radius r is moving in the area between these disks without friction, and is bouncing from disk to disk with elastic collisions. If we choose a small enough r ($r \leq 1/40$ will do) then we can show the following:

Theorem 1. Let $B = \{(0,1),(1,1),...\}$ be the set of 8 lattice points immediately surrounding the origin (0,0). Let $\{\bar{z}_1,...,\bar{z}_n\}$ be any sequence of lattice points which

(1) proceeds by single steps (so that $\bar{z}_k-\bar{z}_{k-1} \in B$ for $k=2,...,n$), and

(2) never proceeds in the same direction for two steps in a row (so that $\bar{z}_k-\bar{z}_{k-1} \neq \bar{z}_{k+1}-\bar{z}_k$ for $k=2,...,n-1$.)

Then we can choose a starting point $\bar{x}_0$ and an initial angle of motion θ_0 so that $\{\bar{z}_1,...,\bar{z}_n\}$ specifies the sequence of fixed disks encountered by the moving disk in its first n bounces.

This theorem is proven in Appendix 1. It shows that the initial angle θ_0 of the moving disk can be chosen so that the disk follows any one of a wide variety of desired paths, even though it also moves strictly in accordance with the classical deterministic laws for frictionless motion and elastic collision. Starting with $\bar{z}_1=(0,0)$, there are 7^{n-1} paths $\bar{z}_1,\ldots,\bar{z}_n$ satisfying conditions (1) and (2) of the theorem. The overwhelming majority of these paths will appear to be completely random in form, but paths can also be specified that express any desired string of coded information. For example, the path $\bar{z}_1,\ldots,\bar{z}_n$ could be chosen so that it spells out Shakespeare's plays in a particular style of handwriting.

Whether the path that is generated is orderly or chaotic, we can look at it as a step by step unfolding in time of information that is stored up in the initial angle θ_0. Indeed, to specify a typical n-step path out of 7^{n-1} paths, we need at least $(n-1)\log_2(7)$ bits of information, and thus we need to specify at least this many bits in the binary expansion of θ_0. We can think of the bouncing disk as steadily reading out and expressing information that is stored up in higher and higher order digits in the binary expansion of θ_0.

We can also look at the path of the disk from the trans-temporal viewpoint adopted here. From this viewpoint, the classical laws of nature are regarded as rules telling how events adjacent in time are connected together. The inherent flexibility of these laws is sufficient to enable the pattern in events unfolding over large time periods to be freely chosen, even though the system seems to evolve by the classical laws over short time periods. In this example, it turns out that the classical laws can be exactly satisfied at all times, even though the large-scale pattern can be chosen with great freedom. In general, however, we require only that the classical laws should be followed closely enough so that they are never violated to an observable degree.

When the disk's path is looked at in this way, the value of θ_0 no longer appears to be as important as it does when it is viewed as the starting point of a chain of causes and effects which completely determines the future of the system. Now the value of θ_0 arises simply as a consequence of the over-all selection of the path. Indeed, if we do not require the classical laws to be strictly obeyed at all times, then the path will not have to be so precisely defined that it forces the binary expansion of θ_0 to encode information for large numbers of successive $\hat{z}_k$'s. In this case only a few digits of this expansion will be significant.

Thus far we have given examples only of systems with a small number of degrees of freedom. However, the ideas that we have illustrated can also be applied to highly complex systems. We suggest that the trajectory followed by a general classical system of n bodies can be viewed mathematically as a flexible continuum which can be bent into a large variety of complex shapes for which the classical laws of motion are nearly obeyed at all times. In situations in which the classical trajectory is highly predictable, this process of selection can produce only trajectories that are very close to exact classical solutions with S=0. This will be true, for example, for exactly integrable systems. However, in situations amenable to deterministic chaos the process of selection will allow for nearly lawful trajectories of highly arbitrary complex form.

The phenomenon of turbulence in fluid flow provides an example of a complex system that appears to exhibit deterministic chaos. Empirical and numerical studies suggest that the large scale motion of the vortices in turbulent flow is strongly effected by the amplification of extremely small variations in the current, and thus the behavior of these vortices is inherently unpredictable.[9] In this situation $NC(\epsilon)$ should contain paths representing many different patterns of turbulent flow, and the process of trans-temporal selection can be viewed as randomly picking out one of these paths.

4. STATISTICAL LAWS

In principle, the process of trans-temporal selection can generate paths in which highly ordered structures seem to arise spontaneously from a state of initial disorder. Yet we normally do not expect to see this, and we expect instead to observe the appearance of a great deal of apparently random noise. This tendency for disorder to increase at the expense of order defines the direction of the passage of time, and as is well known, it is not a consequence of the time reversible laws of motion.

In mathematical models of physical systems this tendency is normally introduced by adding probabilistic laws to the original laws of motion. For example, this is done in statistical mechanics by defining the initial conditions of the system by means of probability distributions known as thermodynamic ensembles. In quantum mechanics we also see that the element of randomness has to be added to the deterministic equation of motion (the Schrödinger equation) as an independent statistical law (von Neumann's postulate of wave function collapse).(10)

The simplest way of introducing thermodynamic randomness into the process of trans-temporal selection is simply to postulate that this process chooses a path "at random" with a probability density proportional to

$$\mu(\bar{Q}) = F(\bar{Q})T[S(\bar{Q})<\epsilon] \qquad (7)$$

where T[condition] is 1 if the condition is true and 0 if it is false.

The function $F(\bar{Q})>0$ places a limitation on the possible paths. For example, if F restricts the paths at times near some t_* in the remote past to a certain highly ordered state of affairs, then trans-temporal selection according to Eqn. (7) should produce a path in which this situation gradually deteriorates from that time onward, and time's arrow goes from past to future. In contrast, if F restricts the paths at times near some t_* in the remote future to a highly ordered state (and imposes no restrictions at other times), then the selection process should produce a path for which time's arrow proceeds from future to past. Of course, we normally expect the first of these two situations to prevail. However, there is some empirical evidence suggesting a significant role for more complex F's, which impose restrictions on the paths at various times. This is discussed in Section 9.

5. QUANTUM MECHANICS

The history of the quantum theory has been marked by a chronic controversy over what is generally known as the quantum mechanical measurement problem. As we shall see, the trans-temporal approach to the laws of physics provides a new way of tackling this problem. We will therefore give a brief description of some of the fundamental issues involved in this controversy and mention some of the main approaches that have been taken to resolve it.

Broadly speaking, there are two ways in which a mathematical theory can describe nature. These are:

(1) The theory quantitatively predicts the patterns of correlation in experimental measurements.

(2) The theory contains mathematical structures which (in principle) represent "objective reality" in a one-to-one fashion.

Here "objective reality" may refer to the totality of what exists in nature, or it may refer to certain limited aspects of nature that a particular theory is intended to describe.

Since acts of measurement are part of reality, a theory satisfying (2) must also satisfy (1). Up to the close of the nineteenth century, it was generally taken for granted that a well developed physical theory should satisfy both (1) and (2). However, the striking feature of quantum mechanics is that while it satisfies (1), it has proven very difficult to find a formulation of the theory which could satisfy criterion (2).

The reason for this is that the state vectors (or wave functions) used to describe nature in quantum mechanics are capable of simultaneously describing many macroscopically distinct states of affairs. There are many situations in which a state vector evolving in accordance with the Schrodinger equation will naturally develop such macroscopic ambiguities. The most famous example is Schrödinger's cat paradox, in which the state vector comes to describe a cat which is simultaneously dead and alive. In quantum mechanics the state vector provides the most complete description of the system that is possible, and thus if such macroscopically ambiguous state vectors are allowed, then it follows that the theory does not give a one-to-one representation of physical reality as we normally conceive of it.

The standard approach to quantum mechanics is to simply accept this state of affairs and adopt (1) as the ultimate criterion for a successful physical theory. According to this approach, it is inherently impossible for a physical theory to provide an intelligible picture of the reality underlying experimental observations. Wigner[6] calls this "the most natural epistemology of quantum mechanics," and it is hard to find fault with it logically or empirically.

However, many attempts have been made to modify or reinterpret quantum mechanics in such a way as to obtain a theory satisfying criterion (2). These include:

(1) Theories involving the automatic "collapse" of the wave function. Here the idea is that as the wave function spreads out through configuration space, it is repeatedly replaced by a modified wave function that is not macroscopically ambiguous. This mathematical procedure of collapse is thought to correspond to an actual process occuring in nature. Some authors have thought that this process is implicit in the formalism of quantum mechanics as it stands,[11] but others have argued that this is not so.[12] A number of physicists have proposed that the collapse of the state vector is connected with the making of an observation by a conscious observer,[2-4] and Wigner in particular has suggested that the laws of quantum mechanics as we now know them will have to be completely reformulated in order to take into account the existence of consciousness as a real feature of nature.

(2) "Hidden variables" theories. Here new parameters are added to the state vector formalism of quantum mechanics to bring it into one-to-one correspondence with reality as we normally conceive of it. A prominent example is the "quantum potential"

theory of David Bohm[13]; a very similar theory is provided by the stochastic mechanics of Edward Nelson.[14]

(3) The "many worlds" (or EWG) theory of Everett, Wheeler, Graham.[15] Here a state vector is posited for the universe as a whole, and this state vector is taken as a direct, one-to-one representation of objective reality. The many macroscopically distinct components of this state vector are interpreted as distinct universes which exert no measurable influence on one another, and thus our idea of objective reality is extended in such a way as to conform with the quantum mechanical formalism.

In the next section we will show that the trans-temporal approach to the laws of physics can be used to develop an additional formulation of quantum mechanics which satisfies criterion (2) for a physical theory.

6. A TRANS-TEMPORAL FORMULATION OF QUANTUM MECHANICS

Our point of departure for reformulating the quantum theory is provided by the Feynman path integral. In the usual formulation of quantum mechanics, the state vector (or wave function) can be thought of as a wave propagating through a multi-dimensional space of parameters. Speaking loosely, the intensity of this wave at one point in space and time can be expressed as a sum of effects derived from nearby points on the wave at a slightly earlier time. Each of these effects can in turn be expressed as a sum of effects deriving from nearby points at a still earlier time, and so on for earlier and earlier times. (This is essentially Huygens' principle.) Thus we can view the wave at one space-time point as being a sum of contributions made along the many different paths leading up to that point.

Feynman[16,17] expresses this idea through the following formula, which he calls a path integral:

$$K_C(t_b,\bar{z}_b;t_a,\bar{z}_a) = \int_C \exp[iA(\bar{z})/\hbar]\, D\bar{z} \tag{8}$$

Here t_a and t_b are two successive times, and $\bar{z}_a$ and $\bar{z}_b$ are two values for the configuration of the system at those times. The parameter z represents a function $\bar{z}(t)$ defining a path. $A(\bar{z})$ is the action for the path $\bar{z}$, as defined in Eqn. (1), and C is a set of paths from $(t_a,\bar{z}_a)$ to $(t_b,\bar{z}_b)$ defining the range of the integration. If C is the set of all paths spanning these limits, the integral can be viewed as the value of a quantum wave function at time t_b and location $\bar{z}_b$, given that the wave function evolved according to the Schrodinger equation corresponding to the action A, and that it was concentrated at $\bar{z}_a$ at time t_a. (We note that it is difficult to define $D\bar{z}$ as a mathematical measure, but rigorous mathematical definitions of the path integral have been made. For example, see DeWitt.)[18]

For some paths $\bar{z}$, the action $A(\bar{z})$ will be sensitive to small variations in $\bar{z}$, and thus the sum of terms $\exp(iA(\bar{z})/\hbar)$ over these nearby paths will tend to cancel out. For other paths, this sensitivity will be less, and the corresponding sum over nearby paths will take on a non-zero value. Thus we can see that the path integral will tend to give weight to paths that approximate the criterion of stationary action that is used to define the classical trajectory of a system. Indeed, the difference between classical and quantum mechanics can be seen as the consequence of allowing many nearly stationary paths, rather than a single exactly stationary path.

Our reformulation of quantum mechanics is based on the following idea: Let us suppose that a physical system actually follows a fuzzy trajectory, called a channel, consisting of a set of similar paths. Feynman argues that the (unnormalized) probability density that the path followed by a quantum mechanical system lies in such a channel should be given by $|K_C(t_b,\bar{z}_b;t_a,\bar{z}_a)|^2$, where the C in Eqn. (8) is now confined to the set of paths constituting this channel.[17] Let us therefore suppose that nature somehow selects a channel C, and that this is done at random in accordance with the probability density given by $|K_C|^2$.

We will refer to the selection of a particular channel in nature as the process of trans-temporal selection. If this selection is performed in accordance with the probabilities $|K_C|^2$ defined by Eqn. (8), then the channel that is selected should represent a sequence of events that conforms with the statistical laws of quantum mechanics.

As we observed in the discussion of Eqn. (7), these probabilities can also be modified by multiplying them by a function $F(C)$ which picks out certain channels in preference to others. This gives us the following quantum mechanical version of Eqn. (7).

$$\mu(C) = F(C)|K_C(t_b,\bar{z}_b;t_a,\bar{z}_a)|^2 \tag{9}$$

By introducing such an F, it is possible to define the arrow of time for the system in the same way that we did this in Section 4 for classical mechanics.

This formulation of quantum mechanics corresponds quite closely to our reformulation of classical mechanics. We can see this by applying it to a simple classical system with n degrees of freedom. Let a be a fixed positive number, and for $i=1,\ldots,n$ let $c_i(t)$ be a real function on the interval from t_a to t_b. The function $\bar{c}(t)=(c_1(t),\ldots,c_n(t))$ defines a path through the n-dimensional configuration space of the system, and we can define a channel C to be the set of all paths $\bar{z}(t)$ satisfying

$$c_i(t)-a \leq z_i(t) \leq c_i(t)+a \tag{10}$$

for $t_a \leq t \leq t_b$ and $i=1,\ldots,n$. (For the path integral to be nonzero we require that the endpoints $\bar{z}_a$ and $\bar{z}_b$ of the paths $\bar{z}(t)$ must lie within the channel.)

Such a channel and one of its enclosed paths is shown in Fig. 3. For simplicity we shall deal with a system defined by the Lagrangian $L = m|\dot{\bar{z}}|^2/2 - V(\bar{z},t)$, where $V(\bar{z},t)$ is a (possibly) time dependent potential. We will denote the path integral in

Eqn. (8) as $K_{V,C}(\bar{z}_b;\bar{z}_a)$, where the parameters t_a and t_b have been left implicit, and the subscript V has been added to express dependence on the potential V.

$K_{V,C}(\bar{z}_b;\bar{z}_a)$ can be thought of as expressing the degree to which a wave function concentrated at $\bar{z}_a$ at time t_a is able to traverse the channel and reach $\bar{z}_b$ at time t_b. This depends on the potential V which specifies the physical interactions occurring in the system. For channels conforming to the path(s) that such a wave function will naturally tend to follow, $K_{V,C}(\bar{z}_b;\bar{z}_a)$ will take on a non-zero value, but for channels following completely different paths, it will be zero or nearly zero.

When we speak of a wave "traversing" the channel C, we mean that the wave satisfies the ordinary Schrodinger equation with potential V within the channel, and that the wave is reduced to zero along the boundary of the channel by an additional imaginary potential. Thus the path integral of Eqn. (8) corresponds to a wave function satisfying the equation

$$i\hbar\partial\Psi(\bar{z})/\partial t = (H + \mathcal{V}_C(\bar{z},t))\Psi(z) \tag{11}$$

where $\mathcal{V}_C(\bar{z},t) = 0$ when $|z_i - c_i(t)| < a$ for $i=1,\ldots,n$, and $\mathcal{V}_C(\bar{z},t) = -i\infty$ otherwise.

We define the free propagation value for a channel of width 2a to be $K_{0,0}(\bar{\xi}_b;\bar{\xi}_a)$. This expression gives the degree to which a wave function concentrated at $\bar{\xi}_a$ at time t_a will be able to freely propagate ($V = 0$) through a straight channel ($c_i = 0$), and reach $\bar{\xi}_b$ at time t_b. If we disregard the possibility of a potential that focuses the wave function within a given channel, we can see that $K_{0,0}(\bar{\xi}_b;\bar{\xi}_a)$ represents the maximum level of wave transmission that we can expect through a channel of width 2a and time span t_b-t_a.

In general, in a quantum mechanical system it will not be possible for $|K_{V,C}(\bar{c}_b+\bar{\xi}_b;\bar{c}_a+\bar{\xi}_a)|$ to be as large as $|K_{0,0}(\bar{\xi}_b;\bar{\xi}_a)|$. This is due to the fact that the potential may cause the wave function to spread out widely, and thus be absorbed by the channel walls much more rapidly than will occur in the case of free propagation. Such spreading will occur, for example, in the situation of Schrödinger's cat, in which the wave function bifurcates into branches representing live and dead cats. The channel will be wide enough to accomodate only one of these alternatives, and the wave representing the other one will be absorbed.

In a system involving only macroscopic variables, however, the wave function may spread out at the rate of a freely propagating wave. The following theorem shows this for the case that we are considering:

Theorem 2. Let $a>0$, the channel C, and the potential V be as defined above. Define

$$s(\bar{c},t,\bar{y}) = |m\ddot{\bar{c}}(t)\cdot\bar{y} + V(\bar{c}(t)+\bar{y},t) - V(\bar{c}(t),t)| \qquad (12)$$

Let $\bar{c}(t_a)=\bar{c}_a$ and $\bar{c}(t_b)=\bar{c}_b$, and assume that $|\xi_{a,i}|\leq a$ for $i=1,\ldots,n$. Then

$$\left[\int_{[-a,a]^n} \left| |K_{V,C}(\bar{c}_b+\bar{\xi}_b;\bar{c}_a+\bar{\xi}_a)| - |K_{0,0}(\bar{\xi}_b;\bar{\xi}_a)| \right|^2 d\bar{\xi}_b \right]^{1/2}$$

$$\leq \exp\left[\int_{t_a}^{t_b} \sup_{\bar{y}} s(\bar{c},t,\bar{y})\, dt/\hbar \right] - 1 \tag{13}$$

where the sup is over $\bar{y}$ for which $|y_i| \leq a$, $i=1,\ldots,n$. This theorem is proven in Appendix 2. It indicates that if a is properly chosen, then a channel following a near-classical path of low sensitivity (given by Eqn. (3)) will conduct a wave function with no greater loss than occurs in a straight channel of width 2a carrying a freely propagating wave. Since we would not generally expect a lower level of loss than this (except perhaps in the case of special, focusing potentials), this means that near-classical paths will tend to be selected by our quantum mechanical process of trans-temporal selection. Since near-classical paths can take on a wide variety of shapes, this means that the paths generated by the quantum mechanical selection process for a "classical" system can also exhibit great variety.

We can illustrate Theorem 2 by applying it to the Henon potential described above. For this potential we can show by a few calculations that

$$\int_{t_a}^{t_b} \sup_{\bar{y}} s(\bar{c},t,\bar{y})dt \le \sqrt{2}a \int_{t_a}^{t_b} |m\ddot{\bar{c}}(t)+\nabla V(\bar{c}(t))|dt$$

$$+ (t_b-t_a)a^2(3+\sqrt{3}+4a/3) \qquad (14)$$

where the sup is over $\bar{y}$ with $|y_i|\le a$, $i=1,2$.

Here the first term on the right hand side is a measure of the deviation of the channel function c(t) from a classical path of the system, and the second term is a constant. The integral in the first term is a measure of how well $\bar{c}(t)$ obeys the classical equation of motion, $m\ddot{\bar{c}}(t)=-\nabla V(\bar{c})$. We have already shown that it is possible for $\bar{c}(t)$ to adhere closely to this equation, even though it deviates considerably from a classical trajectory (for which this term is 0).

Eqn. (14) can be used to estimate an appropriate value for the parameter a defining the width of the channel. If a is too large, then the rightmost term in Eqn. (14) will be large, and we won't be able to conclude that near-classical channels have high quantum mechanical probabilities. On the other hand, if a is too small, then the right hand side of Eqn. (14) will always be small, and all channels will have similar quantum mechanical probabilities (of nearly 0). These considerations lead to the following inequalities for a:

$$\hbar/\sqrt{2} \ll a \ll .45[\hbar/(t_b-t_a)]^{1/2} \qquad (15)$$

where we assume that the path remains within the triangular potential well shown in Fig. 1. For example, if t_b-t_a is 10^{10} seconds, then a should lie between 10^{-27} and 10^{-19} cm.

7. COMPLEX QUANTUM SYSTEMS

Thus far we have considered channels involving functions $c_i(t)$ corresponding to position coordinates z_i for a number of particles. These systems are essentially classical and macroscopic in character. In a general complex system the channel should be defined using macroscopic variables defining positions, densities, or perhaps field strengths. Purely microscopic variables such as electron spin can be left unconstrained by the channel.

An example of a complex system is provided by an elastic medium satisfying the wave equation,

$$\mu\eta_{i,00} - K\eta_{i,jj} = 0, \tag{16}$$

where $\mu=\mu(x,y,z)$ represents the density of the undisturbed medium as a function of position, and K is the medium's modulus of elasticity. Here we use the notation $(t,x,y,z) = (x^0,x^1,x^2,x^3)$, and we adopt the convention that double Latin indices are summed over 1,2,3. A subscript k preceeded by a comma indicates partial differentiation with respect to the corresponding x^k.

In this equation $\overline{\eta}(t,x,y,z)$ represents the displacement at time t of an infinitesimal part of the medium that has an equilibrium position of (x,y,z) and a mass of $\mu(x,y,z)d^3x$. The solutions of the equation represent macroscopic sound waves propagating through the medium. The action is given by

$$A = 1/2 \int d^4x[\mu\eta_{i,0}\eta_{i,0} - K\eta_{i,j}\eta_{i,j}] \qquad (17)$$

and we can define channel functions $c_i(t,x,y,z)$ for $i=1,2,3$. The channel C corresponding to the c_i's is defined to be the set of functions $\eta_i(t,x,y,z)$ for which

$$\|\bar{\eta}(t)-\bar{c}(t)\|_2 \leq \epsilon \qquad (18)$$

for all $t_a \leq t \leq t_b$. Here $\|\cdot\|_2$ is the L_2 norm over x,y,z. This channel consists of functions $\bar{\eta}(t)$ which are macroscopically similar to the channel function $\bar{c}(t)$, but which may differ from it considerably on the microscopic level. Thus the channel prescribes the macroscopic situation, but at the same time allows freedom for microscopic quantum mechanical fluctuations.

We define the path integral over this channel to be

$$K_{V,C}(\bar{\eta}_b;\bar{\eta}_a) = \int_C \exp(iA(\bar{\eta})/\hbar)\ D\bar{\eta} \qquad (19)$$

where $\bar{\eta}$ stands for the functions $\eta_i(t,x,y,z)$, $\bar{\eta}_b$ stands for the functions $\eta_i(t_b,x,y,z)$ for t fixed at t_b, and $\bar{\eta}_a$ stands for the corresponding functions at $t=t_a$. $\bar{\eta}_a$ and

$\bar{\eta}_b$ should satisfy the channel condition of Eqn. (18) at $t = t_a$ and t_b. Here the potential (or potential density) V is defined to be

$$V = K\eta_{i,j}\eta_{i,j} \tag{20}$$

It can be shown that if $\mu(x,y,z)$ varies in a periodic fashion in the x, y, and z directions, then Eqn. (16) will have chaotic solutions. For example, suppose that $\mu(x,y,z)$ takes on high values in spherical regions of macroscopic radius R surrounding the positions (iA,jA,kA), where A>2R and i,j,k vary over the integers. Then, by analogy to the pinball problem, we would expect that small changes in a sound wave at time t will be greatly amplified over a relatively short time. This indicates that there should exist many macroscopically distinct sound wave patterns which are near-solutions of Eqn. (16). If we analyze this problem quantum mechanically using the trans-temporal approach, we can show that there are macroscopically distinct quantum mechanical solutions corresponding to these near-solutions of Eqn. (16). Thus, an elastic medium with a regular pattern of density inhomogenieties should exhibit macroscopic quantum mechanical indeterminism.

This is shown by the following theorem, which is analogous to theorem 2:

THEOREM 3. Given the assumptions and notation introduced thus far, we have

$$\{\int_E [|K_{V,C}(\bar{c}_b+\bar{\xi}_b;\bar{c}_a+\bar{\xi}_a)| - |K_{V,0}(\bar{\xi}_b;\bar{\xi}_a)|]^2 d\bar{\xi}_b\}^{1/2} \leq \tag{21}$$

$$\exp\{\epsilon/\hbar \int_{t_a}^{t_b} dt \|O\bar{c}(t)\|_2\} - 1$$

where the operator $O = \mu(x,y,z)\partial^2/\partial t^2 - K\nabla^2$,

$$\|O\bar{c}(t)\|_2 = [\int d^3x \sum_{i=1}^{3} (\mu c_{i,00} - Kc_{i,jj})^2]^{1/2} \tag{22}$$

and the region of integration in the left hand side of Eqn. (21) is $E = \{\bar{\xi}: \|\bar{\xi}\|_2 \leq \epsilon\}$.

We omit the proof of this theorem since it is closely analogous to the proof of theorem 2. The theorem indicates that if the channel function $c(t,x,y,z)$ nearly satisfies the wave equation (16), then the probability that the corresponding channel C will be selected is comparable with the probability that a straight channel representing an undisturbed medium will be selected.

In this and other similar models we can regard the bundle of paths following the channel as objectively real. The functions defining the channel (such as $\bar{c}(t)$) correspond to the macroscopic aspect of this reality, and the interfering paths in the bundle can be viewed as its purely quantum mechanical aspect. The overall path of the bundle is determined by trans-temporal selection, subject to the requirement that the quantum mechanical laws are closely followed.

Although this model allows us to contemplate an objective picture of physical reality, it also entails a distinction between macroscopic and quantum mechanical levels similar to that stressed by Bohr. Here the macroscopic aspect defined by the channel is not directly tied in with particular acts of measurement as it is in standard presentations of quantum mechanics. However, the values of the macroscopic channel parameters at any given time can be regarded as measurements.

8. RELATIVITY AND NONLOCAL EFFECTS

We have not yet worked out the trans-temporal approach to quantum mechanics for relativistic field theories, but it should be possible to do this in a straightforward fashion. To define a channel we could adopt the approach used with the elastic solid in Section 7 and use functions $c_i(t,x,y,z)$ to specify the macroscopic characteristics of the fields. Trans-temporal selection of the channel can be defined in terms of a field theoretic path integral restricted to paths lying within the channel. The action for such an integral will be relativistically covariant, and the Lorentz transformation of a channel is simply another channel. Thus the trans-temporal selection process will look the same from the viewpoint of any inertial frame.

In this process the channel is selected in such a way as to conform closely to the quantum mechanical laws defined by the particular action functional. As a result, the channel may develop correlations between regions that have a space-like separation. However, this will not violate the relativistic rule that no signal can propagate faster than the speed of light. This rule applies to processes involving cause and effect within space-time, and does not apply to the process of trans-temporal selection which operates globally on the entire space-time situation.

The EPR effect, originally described by Einstein, Podolsky, and Rosen,[19] is a famous example of how a quantum mechanical system can exhibit correlated effects spanning large distances. In the version devised by Bohm[20] two electrons with correlated but otherwise indefinite spins begin to separate at a space-time point A. Later the two electrons pass through space-time points B and C, respectively. When observers at these points measure their spins in a particular direction, they find these spins to be correlated, even though each electron had individually undefined spin. This leads to the rather unexpected conclusion that the two electrons must be treated as a unit, no matter how great their spatial separation may be. The standard quantum mechanical analysis posits a wave function for the two electrons which collapses as a unit, and converts the electrons' initial state of undefined spin into a state with definite, oppositely directed spins.

The same effect is also predicted by the trans-temporal model of quantum mechanics. In this model, however, the collapse of the wave function is replaced by the trans-temporal selection of the channel. The physical set up of the electrons, the observers, and their instruments for measuring spin is such that the chosen channel can bend in one of two ways: with the first electron registering spin up and the second registering spin down, or with the opposite situation. Of course, the physical set up itself is an aspect of the path followed by the channel. Thus, if the channel followed a path in which the instruments measured spin in perpendicular directions, then it would be free to bend in four different ways, representing the four possible independent outcomes of these measurements.

Thus nonlocal effects can be understood as constraints on the ways that the selected channel can bend. We note that the trans-temporal formulation of classical mechanics also entails similar effects. If the chosen trajectory is curved so that particles in one location move in a certain way, then there must be correlated movements at other locations. This is due to the fact that in this trajectory the laws of motion are very nearly followed, and thus movements at separated locations B and C may be mutually influenced by earlier events at A.

9. PARAPSYCHOLOGY

A number of physicists have developed models of parapsychological phenomena based on the collapse of the quantum mechanical wave function.[3-5,21] In these models it is generally proposed that states of conscious volition can influence wave function collapse. This idea can be used to explain how the conscious self can direct brain processes, and it can also be used to account for some parapsychological phenomena such as psychokinesis.

The physicist Helmut Schmidt has performed a number of experiments suggesting that the will of a human observer can influence the outcome of subatomic events such as radioactive decay.[22] In a typical experiment, a random number generator (REG) based on radioactive decay is used to control a display consisting of a number of lights in a ring. Only one light in the ring is lit at any one time, and the apparatus is arranged so that this light will seem to perform a step by step random walk around the circle, with a fifty-fifty chance of going clockwise or counter-clockwise at any one step. Schmidt found in some cases that if a person observing the light desired it to move around the circle in a particular direction, then its random movements would show a small but definite bias in that direction over a considerable period of time. (In other cases the light might show a perverse tendency to move consistently in the opposite direction, and in still others its motion would apparently be completely random.)

Other investigators have observed similar phenomena. For example, a group headed by Robert Jahn of the Princeton School of Engineering has carried out an extensive series of experiments involving REGs driven by microelectronic noise.[23,24] These experiments also showed a consistent correlation between the volition of human observers and deviations in the expected long term behavior of random processes.

A striking feature of these experiments is that the human observer doing the willing has no conscious awareness of how subatomic quantum phenomena are being amplified within the experimental apparatus and used to generate the visible display. Furthermore, these subatomic events may even occur before the observer participates in the experiment and decides to desire a particular pattern of events. Thus Schmidt has obtained positive results from experiments in which the output of the random number generator was recorded on tape and played back later to operate the display.[22] This suggests that the observer's role is simply to desire to see a certain phenomenon, and that physical processes involving quantum mechanical randomness are unfolding teleologically so as to conform with the observer's desire.[24,25]

The trans-temporal approach to quantum mechanics provides a natural explanation for this kind of teleological behavior. To graphically explain this, it is useful to imagine the channel as a kind of flexible rod that can be trans-temporally deformed, but which tends to take on its own preferred shape. Let us suppose that the trans-temporal selection of the channel can be influenced by the will of individual persons. Suppose further that this will acts directly to attain a certain immediate outcome at time t_0 by inducing the channel function $\bar{c}(t)$ to take on values in some appropriate set W at this time. This could be modeled by modifying the function F of Eqn. (9) in such a way as to favor the selection of channels for which $\bar{c}(t_0) \in W$. It follows that if the channel is selected so as to nearly satisfy the physical laws, then the channel function $\bar{c}(t)$ for $t<t_0$ must vary in such a way as to lead up to some point in W. This natural flexing of the channel generates teleological behavior.

If the selected channel represents deterministic, machine-like interactions over the time interval from t_1 to t_2, then that section of the channel will be free to take on a limited variety of shapes, and we can think of it as being inflexible. But if it represents nondeterministic interactions (such as radioactive decay) in the vicinity of t_1, then the channel will be flexible at that point. Thus if an attempt is made to trans-temporally force the channel to take on certain characteristics at time t_2, the result may be the selection of a channel that bends abnormally t_1 so as to allow for this. Thus, if an observer in an REG experiment wills to see certain phenomena at time t_2, it may turn out that radioactive decays violating quantum mechanical statistics occur teleologically at time t_1 in such a way that the deterministic machinery of the experiment can produce the desired result by time t_2.

Effects of this kind do not depend crucially on strictly quantum mechanical phenomena. Sections of the channel corresponding to phenomena of "deterministic chaos" should also be quite flexible, as we argued in Section 3. Indeed, Rhine's classical experiments with dice, and also Jahn's experiments with cascading balls(24) may provide examples of psychokinesis involving deterministic chaos.

Costa de Beauregard(4) has pointed out that if it is possible for a person to psychically influence the EPR spin observations at observation point B, then it follows that the observations at C must be similarly influenced. This allows for communication that could exceed the speed of light. According to the approach taken here, the psychical influences at B can be modeled by constructing F to favor certain configurations at B. As a result, channels with correlated effects at B and C will be preferentially selected, and the person's volition will also effect the situation at C. Since this kind of communication depends solely on trans-temporal effects, it does not violate the theory of relativity, which bars only processes of super-luminal communication based on cause and effect in space-time. We also note that this kind of communication does not depend specifically on quantum mechanical effects. We would also expect it to occur if a classical "chaotic" process at A resulted in correlated effects at B and C, and a psychic was able to influence these effects at B.

10. CONCLUSION

In this paper we have presented a basic framework for theories of physics in which the space-time history of events is seen as an objectively real continuum, and the laws of physics are seen as rules which apply globally to this continuum, constraining its form but not determining that form rigidly. We have applied this framework to both classical mechanics and quantum mechanics, and thus shown how classical mechanics can be endowed with an element of indeterminism, and quantum mechanics can be viewed as describing an observable objective reality.

In the case of quantum mechanics the postulated real continuum consists of a bundle of space-time paths constrained to lie within an envelope that we refer to as a "channel". The channel defines the macroscopic aspects of physical reality, and the paths (or alternatively, waves) within the channel define its microscopic or strictly quantum mechanical aspects. Physically permissible channels are specified by means of probabilities calculated using a Feynman path integral. By this means the laws of physics, expressed by the action functional of the path integral, are used to give a global definition of the channels that are allowed in nature.

In the case of classical mechanics the situation is much simpler. The postulated real continuum consists of a single classical path, and physically permissible paths are defined by the criterion that sensitivity of the action functional to variations in the path should be very small. We show that this definition allows for an enormous variety of paths, even though this sensitivity is made so small that no observable deviations from the classical equations of motion are allowed. Thus classical mechanics can be interpreted as a highly non-deterministc theory, and it also can be concluded that quantum mechanics can be highly non-deterministic when applied to apparently classical systems.

Traditionally, statistical laws have been incorporated in physical theories through the introduction of probabilistic postulates that are added to the laws of motion. Here we also use this approach, and we show that one basic probabilistic postulate can be used for both classical and quantum mechanics. In both cases the laws of motion (defined by the action functional) are used to provide probabilities, and the actual history of events is selected by a random choice in accordance with these probabilities. Since this choice is made once for all of space-time, we refer to it as trans-temporal selection.

To obtain the statistical laws of physics, it is not enough to define possible histories using the action functional, and then choose one at random. For example, this will not provide for the "arrow of time" defining the time direction in which the overall entropy of a system tends to increase. We note that this can be provided for, however, by systematically modifying the probabilities determined by the laws. We do this by multiplying these probabilities by a function F that favors certain possible paths over others.

If such an F is suitably defined, it can cause the selected space-time history to exhibit a variety of effects. For example, it may start out in an ordered state and show progressive increase of disorder; it may show an overall trend from a disordered state to an ordered state; or it may show an overall increase in disorder accompanied by the apparently spontaneous emergence of order at various times. In the latter case there may be teleological behavior, in which apparently random events conspire to produce a later organized event, and there may be non-local effects in which correlated events violating standard statistical laws occur in widely separated regions. These effects can occur in both the classical and the quantum mechanical models, and they depend on the effect of F in modifying the trans-temporal selection process.

Some effects of this kind have been observed in parapsychological experiments in which observers tried to mentally influence quantum mechanical phenomena. According to the trans-temporal selection model, it is possible to account for these experimental results by supposing that individual volition plays a role in the selection process. According to this idea, F should be constructed in such a way as to favor various sets of circumstances that are willed by individuals. The selection process chooses an overall history that blends together the various desired effects and also adheres as closely as possible to the laws of physics.

We should mention that these considerations are consistent with the idea that the trans-temporal selection process is carried out by a transcendental conscious agency that generates the physical world in accordance with basic non-deterministic laws, and the desires of individual, localized conscious entities. This idea has many ramifications, and it has particular relevance to the mind-body problem. It also provides an alternative to the philosophy of deism, which holds that God sustains the universe but plays no active role in the course of material events.

The trans-temporal model of classical and quantum mechanics is intended as a tentative suggestion that may hopefully lead to further insights into the nature of physical laws. In order to fully develop this model in its present form, much mathematical work is needed. For example, the model should be worked out in detail for relativistic field theories, and the proper way to define the channel for general systems should be investigated. It would also be particularly interesting to work out in greater detail the predictions this model makes for various teleological effects. These predictions might provide the basis for further parapsychological experiments which would test the trans-temporal model.

APPENDIX 1. Proof of theorem 1.

Here we begin with the arrangement described in the theorem. Place $\bar{x}_0$ midway between $\bar{z}_1$ and $\bar{z}_2$. The path of the moving disk corresponds to the path of a point which moves on straight lines in the area between the disks of radius 2r centered on the lattice points in the plane, and which reflects from these disks according to the rule that the angle of incidence equals the angle of reflection. Let $[\theta_a,\theta_b]$ be an interval of initial angles for the motion of this point, so that the rays emanating from $\bar{x}_0$ at these angles all strike the disk of radius 2r centered at $\bar{z}_1$, and the rays with angles θ_a and θ_b are tangent to this circle at either side.

We assume that in a grazing collision, the direction of the moving disk is unchanged. After the reflection from the disk at $\bar{z}_1$, the ray will pass through the region outside of the "shadow" S_1 marked in Fig. 4. By picking a suitable $\theta\in[\theta_a,\theta_b]$, this ray can be made to pass through any point in this region. Thus we can narrow down $[\theta_a,\theta_b]$ to an interval $[\theta_a',\theta_b']$ so that if θ lies in this interval, then the ray reflected from disk $\bar{z}_1$ meets disk $\bar{z}_2$, with grazing collisions for $\theta = \theta_a'$ and θ_b'.

After reflecting from disk $\bar{z}_2$, the rays with these initial angles will pass through all points not lying in the shadow S_2 marked in Fig. 4. Since the disk $\bar{z}_3$ does not intersect this shadow, we can narrow down $[\theta_a',\theta_b']$ to an interval $[\theta_a'',\theta_b'']$ so that for θ in this interval, the rays reflected from disk $\bar{z}_2$ will strike disk $\bar{z}_3$, with grazing collisions at $\theta = \theta_a$" and θ_b". The rays reflecting from disk $\bar{z}_3$ will then pass through all points not in the shadow S_3.

In general, we can find a sequence of nested intervals $[\theta_a(k), \theta_b(k)]$ such that the rays with initial θ in this interval strike disks $\bar{z}_1, \ldots, \bar{z}_k$ and precisely bracket disk $\bar{z}_{k+1}$ after reflecting from disk $\bar{z}_k$. This can be done if the radius 2r of the disks is small enough so that the shadow S_k cast by disk $\bar{z}_k$ never intersects any of the 8 disks immediately surrounding $\bar{z}_k$ other than the one opposite disk $\bar{z}_{k-1}$.

By choosing an initial angle of $\theta_0 \in [\theta_a(n-1), \theta_b(n-1)]$ we obtain the path required for the moving disk. Q.E.D.

APPENDIX 2. Proof of theorem 2. Some algebraic manipulations show that

$$K_{V,C}(\bar{z}_b;\bar{z}_a) = \exp[(i/\hbar)(A(\bar{c})+\dot{\bar{c}}_b\cdot\bar{\xi}_b-\dot{\bar{c}}_a\cdot\bar{\xi}_a)]K_{U,0}(\bar{\xi}_b;\bar{\xi}_a) \tag{23}$$

where $\bar{\xi}_a=\bar{z}_a-\bar{c}(t_a)$, $\bar{\xi}_b=\bar{z}_b-\bar{c}(t_b)$, $\dot{\bar{c}}_b=\dot{\bar{c}}(t_b)$, $\dot{\bar{c}}_a=\dot{\bar{c}}(t_a)$, and the potential U is defined by

$$U(\bar{\xi},t) = m\ddot{\bar{c}}(t)\cdot\bar{\xi} + V(\bar{c}(t)+\bar{\xi},t) - V(\bar{c}(t),t) \tag{24}$$

We can express $K_{U,0}(\bar{\xi}_b;\bar{\xi}_a)$ as

$$K_{U,0}(\bar{\xi}_b;\bar{\xi}_a) = (Q\varphi)(\bar{\xi}_b) \tag{25}$$

where φ is the wave function concentrated at the point $\bar{\xi}_a$ (i.e. $\varphi = \delta_{\bar{\xi}_a}$), and Q is the operator for propagation of a wave function from t_a to t_b through a straight channel ($\bar{c} \equiv 0$) using the potential U. We can also consider

$$K_{0,0}(\bar{\xi}_b;\bar{\xi}_a) = (Q_0\Psi)(\bar{\xi}_b) \tag{26}$$

where Q_0 is the corresponding operator for propagation through a straight channel with 0 potential.

Thus the left hand side of Eqn. (15) is

$$\left[\int_{[-a,a]^n} \left||K_{U,0}(\bar{\xi}_b;\bar{\xi}_a)| - |K_{0,0}(\bar{\xi}_b;\bar{\xi}_a)|\right|^2 d\bar{\xi}_b\right]^{1/2} \leq \|Q\Psi - Q_0\Psi\| \tag{27}$$

The operators Q and Q_0 can be approximated as follows. Pick a $j>1$, define $\epsilon = (t_b-t_a)/(j-1)$, and let $t_k=t_a+(k-1)\epsilon$ for $k=1,\ldots,j$. Define the following two operators on functions of $\bar{\xi}$:

$$W = \exp[-(i\hbar\epsilon/2m)\nabla^2] \tag{28}$$

$$D_k(\bar{\xi}) = \exp[-(i\epsilon/\hbar)U(\bar{\xi},t_k)] - 1 \tag{29}$$

where ∇^2 is the n-dimensional Laplacian with respect to $\bar{\xi}$. Also let the projection operator P be defined by $P(\bar{\xi})=1$ if $|\xi_i|\leq a$ for $i=1,\ldots,n$ and $P(\bar{\xi})=0$ otherwise.

Using these operators we can approximate Q by

$$Q(j) = (I+D_j)PW\ldots(I+D_1)PW \tag{30}$$

We can also approximate Q_0 by $Q_0(j) = (PW)^j$.

Let ϕ be an arbitrary wave function. Then $\|U\phi\| = \|\phi\|$ since U is unitary. Also $\|P\phi\| \leq \|\phi\|$ since P is a projection. Finally, for $\phi_1 = P\phi$, $\|D_k\phi_1\| \leq \delta_k\|\phi_1\|$ where δ_k is the sup of $D_k(y)$ for $-a \leq y_i \leq a$, $i=1,\ldots,n$. By expanding Eqn. (30) and using these relationships, we can show that

$$\|Q(j)\Phi - Q_0(j)\Phi\| \leq [(1+\delta_j)\ldots(1+\delta_1)-1]\|\Phi\| \qquad (31)$$

for any wave function Φ. By taking limits as $j \text{---}{>} \infty$ we obtain

$$\|Q(j)\Psi - Q_0(j)\Psi\| \text{ ---}{>} \|Q\Psi - Q_0\Psi\| \qquad (32)$$

and

$$(1+\delta_j)\ldots(1+\delta_1) \text{ ---}{>} \exp[\int_{t_a}^{t_b} \sup_{\bar{y}}|U(\bar{y},t)|dt/\hbar] \qquad (33)$$

Combining these two results with Eqn's. (27) and (31) we obtain the conclusion of the theorem. Q.E.D.

REFERENCES

1. S.L. Jaki, *The Relevance of Physics*, (Univ. of Chicago Press, 1970)

2. E.P. Wigner, "Physics and the Explanation of Life," *Found. of Phys.* 1, 35 (1970)

3. H.P. Stapp, "Consciousness and Values in the Quantum Universe,' *Found. of Phys.* 15, 35 (1985)

4. O. Costa de Beauregard, "Time Symmetry and Interpretation of Quantum Mechanics," *Found. of Phys.* 6, 539 (1976)

5. R.D. Mattuck and E.H. Walker, "The Action of Consciousness on Matter: A Quantum Mechanical Theory of Psychokinesis," in *The Iceland Papers,* A. Puharich, ed. (Essentia Research Associates, 1979), pp. 111-159.

6. E.P. Wigner, "Epistemological Perspective on Quantum Theory," in *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* C.A.Hooker, ed. (Riedel Pub. Co., 1973)

7. W. Heisenberg, "The Representation of Nature in Contemporary Physics," *Daedalus*, 87, 99 (1958)

8. M. Henon in *Proceedings of the Summer School "Chaotic*

Behavior of Deterministic Systems" (Les Houches, July 1981), G. Iooss, R.H.G. Helleman, R. Stora, eds. (North-Holland, 1983)

9. J.E. Fromm and F.H. Harlow, "Numerical Solution of the Problem of Vortex Sheet Development," *Phys. of Fluids* 6, 975 (1963)

10. J. von Neumann, *Mathematical Foundations of Quantum Mechanics,* (Princeton Univ. Press, 1955), pp. 417-445.

11. A. Daneri, A. Loinger, and G.M. Prosperi, "Further Remarks on the Relations Between Statistical Mechanics and Quantum Theory of Measurement," *Nuovo Cim.* XLIV, 120 (1966)

12. J.M. Jauch, E.P. Wigner, and M.M. Yanase, "Some Comments Concerning Measurement in Quantum Mechanics," *Nuovo Cim.* XLVIII, 144 (1967)

13. D. Bohm and B.J. Hiley, "Measurement Understood Through the Quantum Potential Approach," *Found. of Phys.* 14, 255 (1984)

14. E. Nelson, *Quantum Fluctuations,* (Princeton Univ. Press, 1985)

15. B.S. DeWitt and N. Graham, *The Many-Worlds Interpretation of Quantum Mechanics,* (Princeton Univ. Press, 1973)

16. R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals*, (McGraw Hill, 1965)

17. R.P. Feynman, "Space-Time Approach to Non-Relativistic Quantum Mechanics," *Rev. of Mod. Phys.* 20, 367 (1948)

18. C.M. DeWitt, "Feynman's Path Integral", *Commun. math. Phys.* 28, 47 (1972)

19. A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Reality Be Considered Complete?", *Phys. Rev.* 47, 777, (1935)

20. D. Bohm, *Quantum Theory*, (Prentice Hall, 1951)

21. H. Schmidt, "Collapse of the State Vector and Psychokinetic Effect," *Found. of Phys.* 12, 565 (1982)

22. H. Schmidt, "Evidence for Direct Interaction between the Human Mind and External Quantum Processes," in *Proceedings of the International Conference on Cybernetics and Society*, (IEEE, Inc., 1977)

23. R.G. Jahn and B.J. Dunne, *An REG Experiment with Large Data Base Capability, III: Operator Related Anomalies*, (School of Engineering/Applied Science, Princeton Univ., 1984)

24. B.J. Dunne, R.G. Jahn, and R.D. Nelson, *Princeton Engineering Anomalies Research*, (School of Engineering/Applied Science, Princeton Univ., 1985)

25. R.G. Stanford, "Experimental Psychokinesis: A review from Diverse Perspectives," in *Handbook of Parapsychology*, B. Wolman, ed. (Van Nostrand Reinhold, 1977)

Captions for the figures:

1. Deterministic chaos generated by the Henon potential for a total energy of E=1/6. [This figure is from ref. (8), p 90.]

2. A closed orbit generated by a particle moving in accordance with the Henon potential.

3. An example of a simple quantum mechanical channel, showing one of the system paths which it encloses.

4. Diagram showing the limits of amplification of variations in initial direction for the classical pinball system. See Appendix 1 for details.

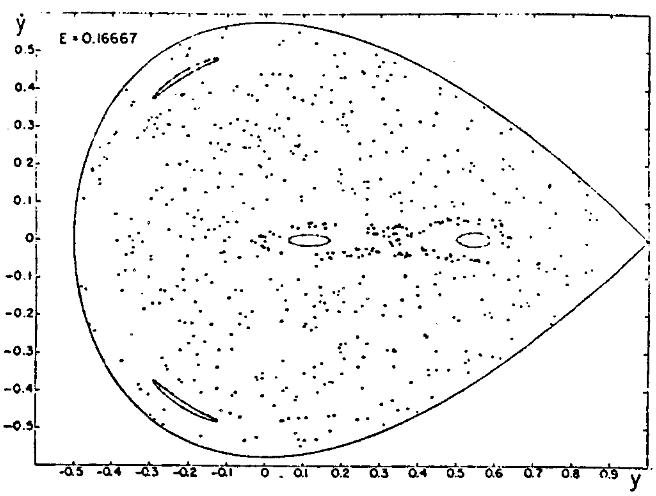
ẏ
ε = 0.16667
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
y

FIG. 1.

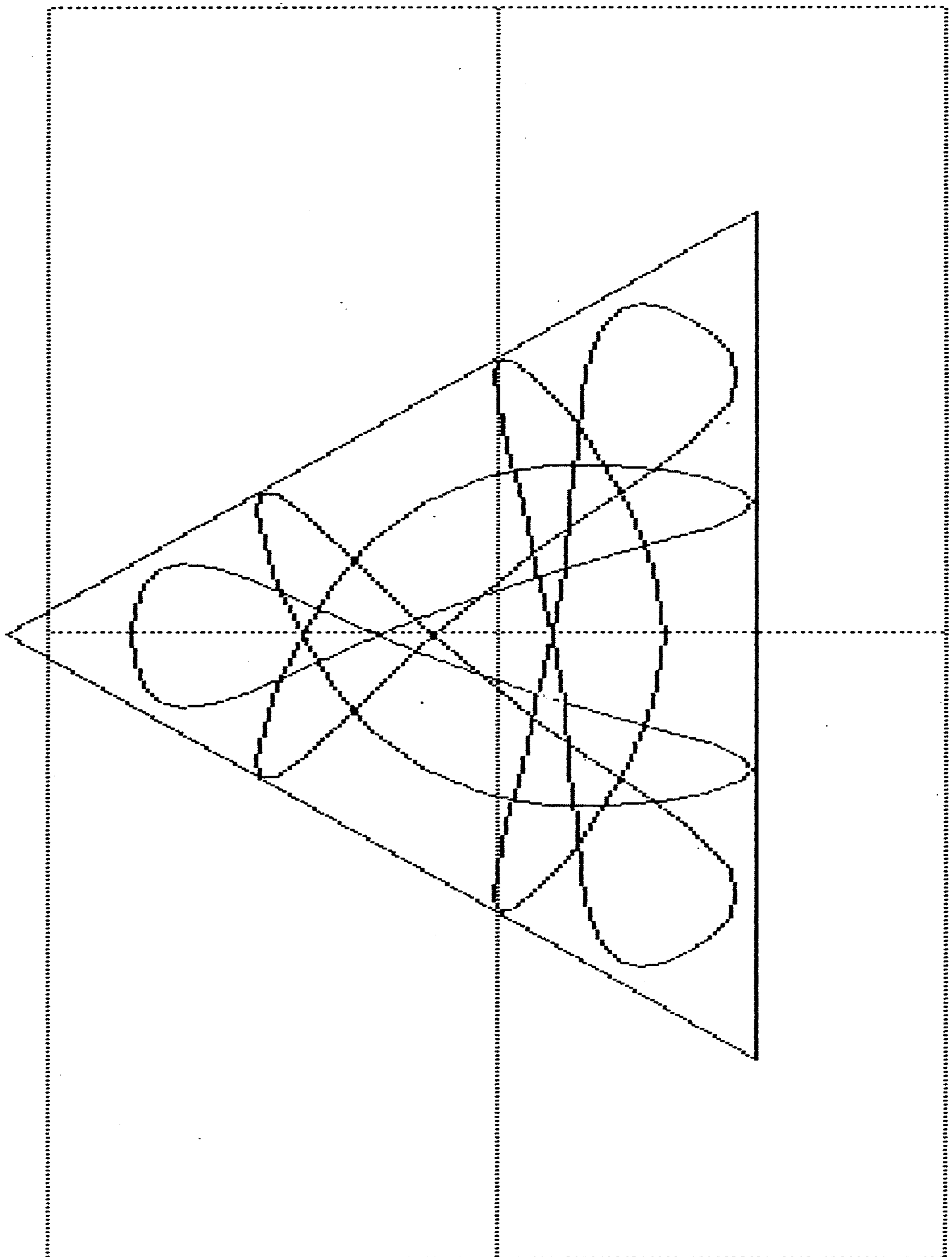

FIG. 2

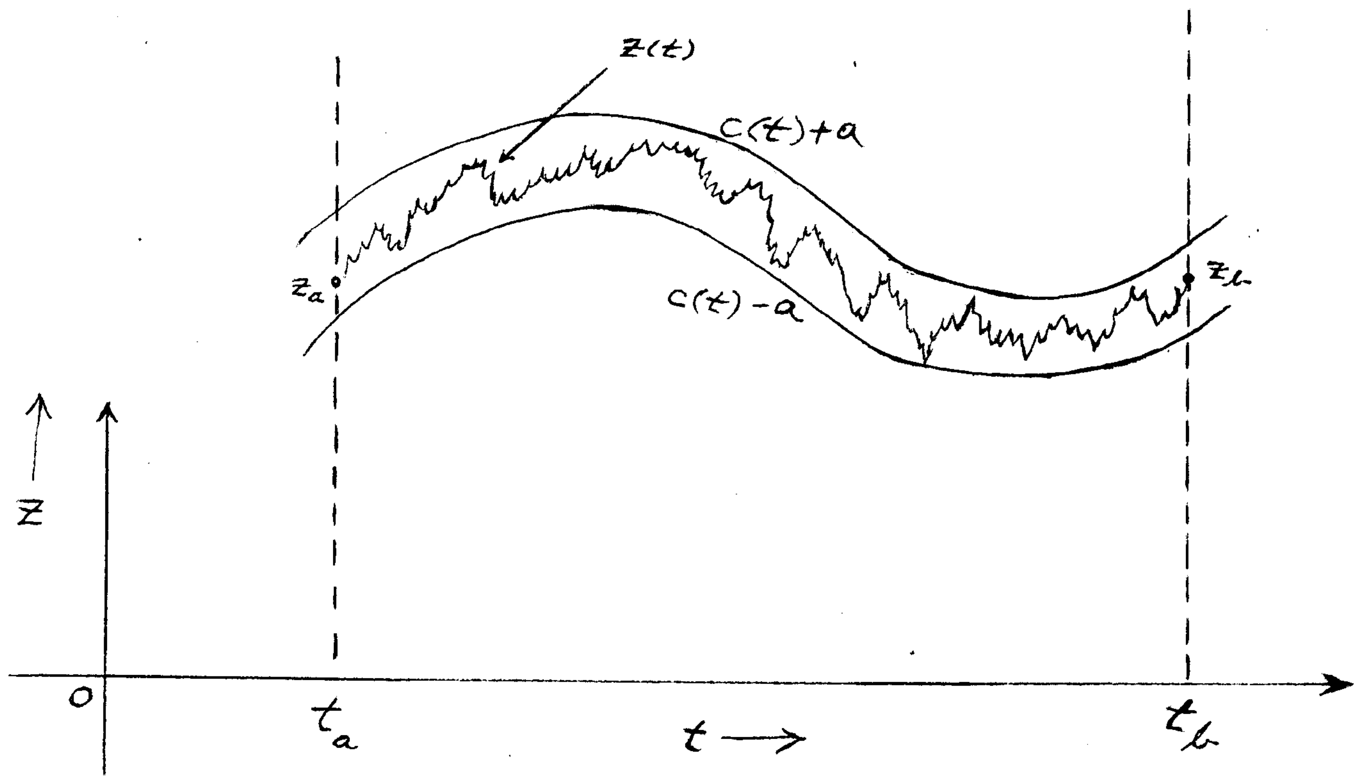

Fig. 3.

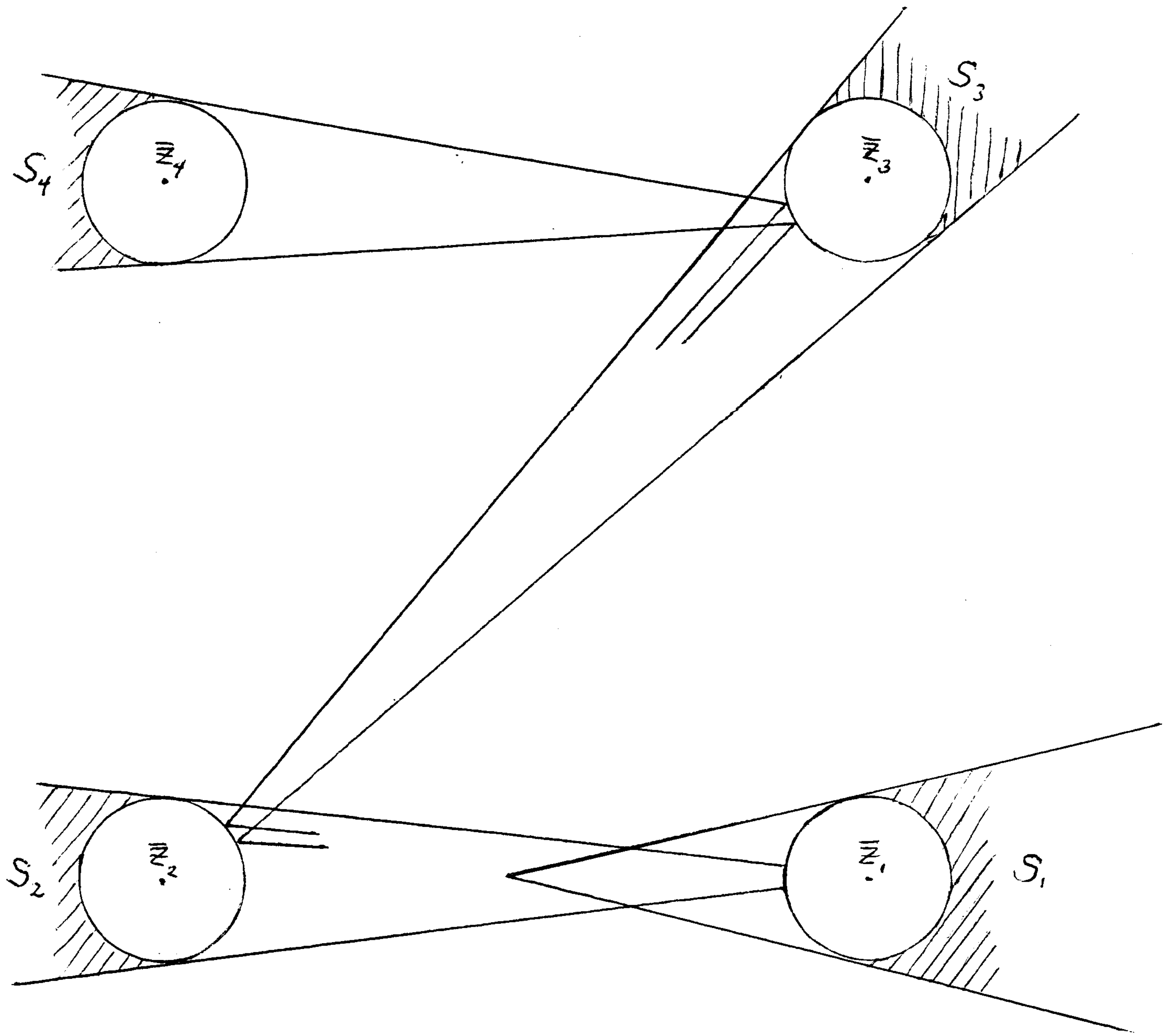

Fig. 4.